2226 – 5015M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2026

Mathematics: analysis and approaches

Higher level

Paper 1

27 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite student errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg **M1**, **A2**.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some students may be advantaged for that specific question item, it is likely that these students will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	***Further working seen***	***Any FT issues?***	***Action***
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	*No.* *Last part in question.*	*Award **A1** for the final mark (condone the incorrect further working)*
2.	$\frac{35}{72}$	*0.468111…* *(incorrect decimal value)*	*Yes.* *Value is used in subsequent parts.*	*Award **A0** for the final mark (and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.

- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word “their” in a description, to indicate that students may be using an incorrect value.

- If the student’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when students fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.

- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.

- If a student makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a student incorrectly copies values or information from the question, this is a mis-read (***MR***). A student should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of students' own work does **not** constitute a misread, it is an error.
- If a student uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Students will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the student work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however students may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Students are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

No calculator is allowed. The use of any calculator on this paper is malpractice and will result in no grade awarded. If you see work that suggests a student has used any calculator, please follow the procedures for malpractice.

10 Presentation of student work

Crossed out work: If a student has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the student indicates that they would like the work to be marked.

More than one solution: Where a student offers two or more different answers to the same question, an examiner should only mark the first response unless the student indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

Section A

1. (a) **METHOD 1**

EITHER

roots are 2 and q ***(A1)***

$\frac{1}{2}(2+q)=4$ ***A1***

OR

Recognises that distance between zeros and axis of symmetry is 2 ***(A1)***

$\frac{q-2}{2}=2$, $4+2=q$, sketch showing 2, 4 and 6 on the x-axis (or number line) ***A1***

THEN

$q=6$ ***AG***

[2 marks]

METHOD 2

expanding brackets

$3(x^2-2x-qx+2q)$ $(=3(x^2-x(2+q)+2q)$ $=3x^2-3x(2+q)+6q)$ ***(A1)***

EITHER

using $\frac{-b}{2a}=4$

$\frac{1}{2}(2+q)=4$, $\frac{3(2+q)}{6}=4$ ***A1***

OR

using $f'(4)=0$

$6x-3(2+q)=0$

$24-3(2+q)=0$ ***A1***

OR

using completing the square

$$3\left[\left(x-\frac{2+q}{2}\right)^2-\left(-\frac{2+q}{2}\right)^2+2q\right]$$

$\frac{1}{2}(2+q)=4$ ***A1***

continued...

Question 1 continued

OR

using vertex form $3(x-a)^2+c$

when $a=4$ $f(x)=3(x^2-8x+16)+c$

$-2-q=-8$ ***A1***

THEN

$q=6$ ***AG***

[2 marks]

(b) **EITHER**

recognition that $f(4)$ is required ***(M1)***

$f(4)=3(4-2)(4-6)$

OR

attempts to complete the square using $f(x)$ ***(M1)***

$3(x-4)^2-12$, $3\left[-\left(-\dfrac{2+6}{2}\right)^2+2\times 6\right]$

OR

attempts to substitute $q=6$ into $3\left[-\left(-\dfrac{2+q}{2}\right)^2+2\times q\right]$ ***(M1)***

$(4,-12)$ ***A1***

Note: Accept $x=4, y=-12$.

[2 marks]

(c) Range: $f(x)\geq -12$ ***A1***

Note: Accept $y\geq -12$ or correct interval notation.

Note: Award follow through for $f(x)\geq$ their min, as long as their min is negative.

[1 mark]

[Total: 5 marks]

2. (a) $\ln(t+1)-\mathrm{e}^{-t}\ (+c)$ ***A1A1***

Note: Award ***A1*** for each correct term.

[2 marks]

(b) **METHOD 1**

recognition that $(s=)\int_0^t v\,dt$ ***(M1)***

$\left[\ln(t+1)-e^{-t}\right]_0^t$

attempt to substitute their limits into their integrated expression and subtracts in any order ***M1***

$\ln(t+1)-e^{-t}-\left(\ln 1-e^{0}\right)$

$\ln(t+1)-e^{-t}-\left(0-1\right)$

$\ln(t+1)-e^{-t}+1$ ***A1***

METHOD 2

recognition that $s=\int v\,dt$ ***(M1)***

attempt to substitute $t=0$ and $s=0$ into their equation to find a value for c. ***M1***

$0=\ln(0+1)-\mathrm{e}^{-0}+c$

Note: Only award **M1** if the constant of integration is present.

$(s=)\ln(t+1)-\mathrm{e}^{-t}+1$ ***A1***

Note: Condone both inclusion and absence of modulus inside the $\ln$ function throughout, as the argument is positive.

[3 marks]
[Total: 5 marks]

3. (a) **METHOD 1**

EITHER

recognition that the ratio between u_8 and u_6 is r^2 ***(M1)***

$u_8 = u_6 \times r^2$, $r^2 = \frac{u_8}{u_6}$

OR

attempt to form a pair of simultaneous equations in terms of u_1 and r ***(M1)***

$u_1 \times r^5 = 1.6 \times 10^5$ and $u_1 \times r^7 = 6.4 \times 10^5$

THEN

correct equation in terms of r or r^2 only ***(A1)***

$6.4 \times 10^5 = 1.6 \times 10^5 \times r^2$, $r^2 = \frac{6.4 \times 10^5}{1.6 \times 10^5}(=4)$, $r^2 = \frac{6.4 \times 10^5}{1.6 \times 10^5}(=4)$

$r = 2$ ***A1***

Note: Do not award final **A1** if $r = -2$ is also given as a solution and not explicitly discarded.

METHOD 2

recognition that $r = \frac{u_7}{u_6} = \frac{u_8}{u_7}$ ***(M1)***

$u_7 = 3.2 \times 10^5$

$r = \frac{6.4 \times 10^5}{3.2 \times 10^5}$, $r = \frac{3.2 \times 10^5}{1.6 \times 10^5}$, $r = \frac{640000}{320000}$, $r = \frac{320000}{160000}$ ***(A1)***

$r = 2$ ***A1***

[3 marks]

(b) valid attempt to find u_1 ***(M1)***

$u_1 \times 2^5 = 1.6 \times 10^5$, $u_1 \times 2^7 = 6.4 \times 10^5$, or equivalent OR repeated division by 2

$u_1 = 5000$, $u_1 = 0.5 \times 10^4$, $u_1 = \frac{6.4 \times 10^5}{2^7}$, $u_1 = \frac{1.6 \times 10^5}{2^5}$, $u_1 = \frac{640000}{128}$, $u_1 = \frac{160000}{32}$ ***(A1)***

$u_1 = 5 \times 10^3$ ***A1***

Note: Accept $a = 5$ and $k = 3$

[3 marks]
[Total: 6 marks]

4. (a) **METHOD 1**

attempt to use the change of base identity $\log_a x = \dfrac{\log_b x}{\log_b a}$ (seen anywhere) ***M1***

attempt to use the power rule $\log_a x^m = m\log_a x$ (seen anywhere) ***M1***

$\log_5 25 = 2$ OR $\log_{25} 5 = \dfrac{1}{2}$ (seen anywhere) ***A1***

correct work leading to:

$\log_5(9x)$, $\dfrac{2\log_5 9 + 2\log_5 x}{2}$ ***A1***

$\log_5 9 + \log_5 x$ ***AG***

[4 marks]

METHOD 2

attempt to use $\log_{a^n} x = \dfrac{1}{n}\log_a x$ (seen anywhere) ***M1***

attempt to use the power rule $\log_a x^m = m\log_a x$ (seen anywhere) ***M1***

$\dfrac{1}{2}\log_5(81x^2)$, $\dfrac{\log_5 81 + \log_5 x^2}{2}$ ***A1***

correct work leading to:

$\log_5(9x)$, $\dfrac{2\log_5 9 + 2\log_5 x}{2}$ ***A1***

$\log_5 9 + \log_5 x$ ***AG***

[4 marks]

METHOD 3

$\log_{25}(81x^2) = a$

$81x^2 = 25^a$ ***M1***

$(9x)^2 = 5^{2a}$ ***M1***

$9x = 5^a$ ***A1***

$a = \log_5 9x$ ***A1***

$\log_5 9 + \log_5 x$ ***AG***

[4 marks]

continued…

Question 4 continued

(b) $\log_5 \sqrt[3]{x} = \log_5 9 + \log_5 x - \log_5 4$

attempt to use addition and subtraction rules for logs to obtain an equation of the form

$\log_5 A = \log_5 B$ ***(M1)***

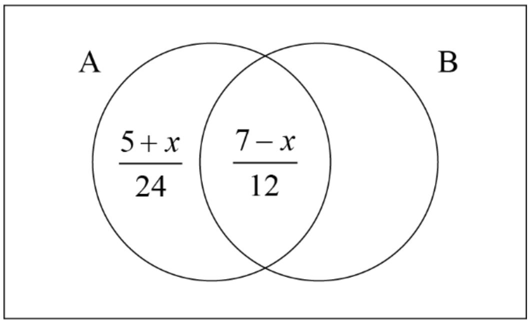
$\log_5 \sqrt[3]{x} = \log_5 \frac{9x}{4}$

$x^{\frac{2}{3}} = \frac{4}{9}$ OR $x^{-\frac{2}{3}} = \frac{9}{4}$ OR $x^2 = \frac{64}{729}$ ***(A1)***

$x = \frac{8}{27}$ ***A1***

[3 marks]
[Total: 7 marks]

5. (a) attempt to draw Venn diagram or attempt to use $\mathrm{P}(A) = \mathrm{P}(A \cap B') + \mathrm{P}(A \cap B)$ ***(M1)***



$\mathrm{P}(A) = \dfrac{5+x}{24} + \dfrac{7-x}{12}$ (***A1***)

$= \dfrac{5+x}{24} + \dfrac{2(7-x)}{24}$ ***A1***

$= \dfrac{19-x}{24}$ ***AG***

[3 marks]

(b) $\mathrm{P}(B)\left(= \dfrac{7-x}{12} + \dfrac{1}{6}\right) = \dfrac{9-x}{12}$ (seen anywhere) ***(A1)***

attempt to use $\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B)$ to form equation in x: ***(M1)***

$\dfrac{7-x}{12} = \left(\dfrac{5+x}{24} + \dfrac{7-x}{12}\right)\left(\dfrac{7-x}{12} + \dfrac{1}{6}\right)$, $\dfrac{7-x}{12} = \left(\dfrac{19-x}{24}\right)\left(\dfrac{9-x}{12}\right)$ ***(A1)***

$168 - 24x = x^2 - 28x + 171$

$x^2 - 4x + 3 = 0$, $12x^2 - 48x + 36 = 0$ ***A1***

attempt to solve their quadratic equation ***(M1)***

$x = 1,\ x = 3$ ***A1***

[6 marks]

[Total: 9 marks]

6. METHOD 1

attempt to express unshaded area as difference of two sectors (with any angle) ***(M1)***

unshaded area = $\frac{1}{2}\times r^2\times\frac{5\pi}{3}-\frac{1}{2}\times 6^2\times\frac{5\pi}{3}\left(=\frac{5r^2\pi}{6}-30\pi\right)$ or $\frac{1}{2}\times r^2\times\text{their }\theta-\frac{1}{2}\times 6^2\times\text{their }\theta$

attempt to set unshaded area to $\frac{2}{3}$ of the shaded area (with any angle) ***(M1)***

$\dfrac{\frac{1}{2}\times r^2\times\text{their }\theta-\frac{1}{2}\times 6^2\times\text{their }\theta}{\frac{1}{2}\times 6^2\times\text{their }\theta}=\frac{2}{3}$ or $\frac{1}{2}\times r^2\times\text{their }\theta-\frac{1}{2}\times 6^2\times\text{their }\theta=\frac{2}{3}\times\frac{1}{2}\times 6^2\times\text{their }\theta$

Note: The second Method mark depends on the first one so do not award ***(M0)(M1)***

$\dfrac{\frac{1}{2}\times r^2\times\frac{5\pi}{3}-\frac{1}{2}\times 6^2\times\frac{5\pi}{3}}{\frac{1}{2}\times 6^2\times\frac{5\pi}{3}}=\frac{2}{3}\left(\dfrac{\frac{5r^2\pi}{6}-30\pi}{30\pi}=\frac{2}{3}\right)$ or $\frac{1}{2}\times r^2\times\frac{5\pi}{3}-\frac{1}{2}\times 6^2\times\frac{5\pi}{3}=20\pi$ or equivalent ***A1***

$\frac{r^2-6^2}{6^2}=\frac{2}{3}\left(\frac{r^2}{36}-1=\frac{2}{3}\right)$

$r=\sqrt{60}$ ***A1***

METHOD 2

recognise that the shaded area is $\frac{3}{5}$ of the total area ***(M1)***

$\frac{3}{5}$ or $\frac{5}{3}$ seen as ratio of areas

attempt to express shaded area as $\frac{3}{5}$ of the total (with any angle) ***(M1)***

$\dfrac{\frac{1}{2}\times 6^2\times\text{their }\theta}{\frac{1}{2}\times r^2\times\text{their }\theta}=\frac{3}{5}$ or $\frac{1}{2}\times 6^2\times\text{their }\theta=\frac{3}{5}\times\frac{1}{2}\times r^2\times\text{their }\theta$ or $\frac{5}{3}\times\frac{1}{2}\times 6^2\times\text{their }\theta=\frac{1}{2}\times r^2\times\text{their }\theta$

Note: The second Method Mark depends on the first one so do not award ***(M0)(M1)***

$\dfrac{\frac{1}{2}\times 6^2\times\frac{5\pi}{3}}{\frac{1}{2}\times r^2\times\frac{5\pi}{3}}=\frac{3}{5}\left(\dfrac{30\pi}{\frac{5r^2\pi}{6}}=\frac{3}{5}\right)$ or $\dfrac{\frac{1}{2}\times 6^2\times\frac{5\pi}{3}}{\frac{1}{2}\times r^2\times\frac{5\pi}{3}}=\frac{3}{5}\left(30=\frac{r^2}{2}\right)$ or equivalent ***A1***

continued...

Question 6 continued

$$\frac{6^2}{r^2} = \frac{3}{5}$$

$$r = \sqrt{60}$$ ***A1***

[Total: 4 marks]

7. (a) Let $A = \text{"is pregnant"}$ and $B = \text{"produces a positive test result"}$.

valid initial approach for finding $\mathrm{P}(B)$ ***(M1)***

attempt to use conditional probability to find $\mathrm{P}(A \cap B)$ and $\mathrm{P}(A' \cap B)$ OR drawing a probability tree diagram

$\mathrm{P}(B) = 0.4 \times 0.9 + 0.6 \times 0.2$ ***(A1)***

$= 0.48 \ \left(= \frac{12}{25}\right)$ ***A1***

[3 marks]

(b) attempt to use conditional probability formula in context ***(M1)***

$\mathrm{P}(\text{pregnant} \mid \text{positive}) = \frac{\mathrm{P}(\text{pregnant } \cap \text{ positive})}{P(\text{positive})}$

$= \frac{0.36}{0.48}$ ***(A1)***

$= 0.75 \ \left(= \frac{3}{4}\right)$ ***A1***

[3 marks]

[Total: 6 marks]

8. (a) (i) other two roots are $a+\mathrm{i},\ 2-b\mathrm{i}$ ***(A1)***

$\frac{7}{3}=\frac{1}{3}+(a+\mathrm{i})+(a-\mathrm{i})+(2-b\mathrm{i})+(2+b\mathrm{i})$ OR $\frac{7}{3}=\frac{1}{3}+2a+4$ ***A1***

$a=-1$

AG
[2 marks]

(ii) attempt to use formula for product of 5 roots: ***(M1)***

$\frac{26}{3}=\frac{1}{3}(-1+\mathrm{i})(-1-\mathrm{i})(2+b\mathrm{i})(2-b\mathrm{i})$

$26=2\left(4+b^2\right)$ ***(A1)***

$b=3$ ***A1***

[3 marks]

(b) $p(z)=3\left(z-\frac{1}{3}\right)\left(z-(-1+i)\right)\left(z-(-1-i)\right)\left(z-(2+3i)\right)\left(z-(2-3i)\right)$

$=(3z-1)\left(z^2+2z+2\right)\left(z^2-4z+13\right)$ ***A1A1A1***

Note: Accept $(A=3,\ B=-1)(C=2, D=2)(E=-4, F=13)$ or
$(A=3,\ B=-1)(E=2, F=2)(C=-4, D=13)$

Note: Award **A1** for each correct factor or each correct pair of values.
Award ***A0*** for the first factor if there is a linear factor of $\frac{1}{3}$ given in final answer.

[3 marks]
[Total: 8 marks]

9. (a) **EITHER**

$\mathbf{v} \bullet \mathbf{a} = (\mathbf{a} \bullet \mathbf{c})(\mathbf{b} \bullet \mathbf{a}) - (\mathbf{a} \bullet \mathbf{b})(\mathbf{c} \bullet \mathbf{a})$ *A1*

$\mathbf{a} \bullet \mathbf{b} = \mathbf{b} \bullet \mathbf{a}$ and $\mathbf{a} \bullet \mathbf{c} = \mathbf{c} \bullet \mathbf{a}$ *R1*

OR

$\mathbf{a} \bullet \mathbf{v} = (\mathbf{a} \bullet \mathbf{c})(\mathbf{a} \bullet \mathbf{b}) - (\mathbf{a} \bullet \mathbf{b})(\mathbf{a} \bullet \mathbf{c})$ *A1*

$\mathbf{v} \bullet \mathbf{a} = \mathbf{a} \bullet \mathbf{v}$ (seen anywhere) *R1*

Note: DO NOT AWARD *A0R1*

$\mathbf{v} \bullet \mathbf{a} = 0$ ***AG***

[2 marks]

(b) (i) $\mathbf{b} \times \mathbf{c}$ is perpendicular/orthogonal to $\mathbf{b}$ ***R1***

[1 mark]

(ii) **METHOD 1**

$\mathbf{v} \bullet (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \bullet \mathbf{c})(\mathbf{b} \bullet (\mathbf{b} \times \mathbf{c})) - (\mathbf{a} \bullet \mathbf{b})(\mathbf{c} \bullet (\mathbf{b} \times \mathbf{c}))$ ***A1***

$\mathbf{b} \times \mathbf{c}$ is perpendicular/orthogonal to $\mathbf{c}$ ***(R1)***

$\mathbf{c} \bullet (\mathbf{b} \times \mathbf{c}) = 0$ (seen anywhere) ***A1***

$\mathbf{v} \bullet (\mathbf{b} \times \mathbf{c}) = 0$ ***AG***

Note: Do not award ***A0(R1)A1*** or ***A0(R1)A0***.

[3 marks]

METHOD 2

$\mathbf{v}$ is a linear combination of $\mathbf{b}$ and $\mathbf{c}$ (may be implied by sketch of plane containing $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{v}$) ***R1***

$\mathbf{b} \times \mathbf{c}$ is perpendicular/orthogonal to $\mathbf{c}$ ***(R1)***

$\mathbf{v}$ is perpendicular/orthogonal to $\mathbf{b} \times \mathbf{c}$ ***A1***

$\mathbf{v} \bullet (\mathbf{b} \times \mathbf{c}) = 0$ ***AG***

Note: Do not award ***R0(R1)A1*** or ***R0(R1)A0***.

[3 marks]

(c) $\mathbf{v}$ is perpendicular/orthogonal to $\mathbf{a}$ and $\mathbf{b} \times \mathbf{c}$ ***R1***

hence $\mathbf{v}$ is parallel to $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$ (accept $v = \mathbf{a} \times (\mathbf{b} \times \mathbf{c})$) ***A1***

Note: Award **R0A1** for correct answer without justification.

[2 marks]
[Total: 8 marks]

Section B

10. (a) finds $f''(x)$ (seen anywhere)

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$ ***(A1)***

recognises that $f''(-2) = 0$ ***(M1)***

$6a \times -2 + 2b = 0$

$-12a + 2b = 0$ or equivalent ***A1***

equate $f(-2)$ to the y-coordinate (seen anywhere)

$a(-2)^3 + b(-2)^2 + c(-2) + d = -2c + d + 24$

$-8a + 4b - 2c + d = -2c + d + 24 \quad (-2a + b = 6)$ ***A1***

Note: Allow recovery from missing brackets.

valid attempt to solve their two linear equations in 'a' and 'b' only ***(M1)***

correct work leading to a value of 'a', which is then used to find a value for 'b', or vice versa ***A1***

$a = \frac{3}{2},\ b = 9$ ***AG***

[6 marks]

(b) (i) $f'(x) = \frac{9}{2}x^2 + 18x + c$

recognize that $f'(-3) = 0$ ***(M1)***

$\frac{81}{2} - 54 + c = 0$ ***A1***

$c = \frac{27}{2}$ ***AG***

[2 marks]

(ii) $f'(k) = 0$

$\frac{9}{2}k^2 + 18k + \frac{27}{2} = 0 \quad \left(\Rightarrow k^2 + 4k + 3 = 0\right)$ ***(A1)***

valid attempt to solve their quadratic ***(M1)***

$(k+1)(k+3) = 0$ OR $k = \frac{-4 \pm \sqrt{4^2 - 4(1)(3)}}{2(1)}$ OR $(k+2)^2 - 1 = 0$

$k = -1$ ***A1***

Note: Condone working with x instead of k throughout. Award full marks for an answer of $k = -1$ from considering symmetry of roots around point of inflexion.

[3 marks]

continued...

Question 10 continued

(iii) **EITHER**

$f''(-1) = \left(9(-1)+18=9\right) > 0$ ***A1***

OR

$f'(x)$ is negative to the left, and positive to the right, of $f'(-1)$ ***A1***

Note: If a table or line is used, $f'(x)$ must be seen

OR

sketch of positive cubic with x-values of stationary points ***A1***

THEN

(local) minimum at $x=-1$ ***A1***

Note: Do not award ***A0A1***. Do not award **FT** in this sub-part.

[2 marks]

(c) recognition that $f(-3)$ is required ***(M1)***

$f(-3) = \frac{3}{2}(-3)^3 + 9(-3)^2 + \frac{27}{2}(-3) + d$

$f(-3) = d$ ***A1***

tangent passes through P ***AG***

[2 marks]

[Total: 15 marks]

11. (a) **METHOD 1**

EITHER

$$\cot 2\theta \equiv \frac{1}{\tan 2\theta}\left(\equiv \frac{1}{\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)}\right)$$ ***(A1)***

$$\equiv \frac{1-\tan^2\theta}{2\tan\theta}$$ ***A1***

$$\equiv \frac{1}{2\tan\theta}-\frac{\tan\theta}{2} \quad \text{OR} \quad \frac{1}{2}\left(\frac{1}{\tan\theta}-\tan\theta\right) \text{ OR } \frac{\cot^2\theta-1}{2\cot\theta}$$ ***A1***

$$\equiv \frac{1}{2}\left(\cot\theta-\frac{1}{\cot\theta}\right)$$ ***AG***

OR

$$\frac{1}{2}\left(\cot\theta-\frac{1}{\cot\theta}\right)\equiv\frac{1}{2}\left(\frac{1}{\tan\theta}-\tan\theta\right) \text{ OR } \frac{1}{2\tan\theta}-\frac{\tan\theta}{2}$$ ***A1***

$$\equiv \frac{1-\tan^2\theta}{2\tan\theta}$$ ***A1***

$$\equiv \frac{1}{\tan 2\theta}$$ ***A1***

$$\equiv \cot 2\theta$$ ***AG***

[3 marks]

METHOD 2

EITHER

$$\cot 2\theta \equiv \frac{\cos 2\theta}{\sin 2\theta}$$ ***(A1)***

$$\equiv \frac{\cos^2\theta-\sin^2\theta}{2\sin\theta\cos\theta}\left(\equiv\frac{\cos^2\theta}{2\sin\theta\cos\theta}-\frac{\sin^2\theta}{2\sin\theta\cos\theta}\equiv\frac{1}{2}\left(\frac{\cos^2\theta}{\sin\theta\cos\theta}-\frac{\sin^2\theta}{\sin\theta\cos\theta}\right)\right)$$ ***A1***

$$\equiv \frac{\cos\theta}{2\sin\theta}-\frac{\sin\theta}{2\cos\theta}$$ ***A1***

$$\equiv \frac{1}{2}\left(\cot\theta-\frac{1}{\cot\theta}\right)$$ ***AG***

continued…

Question 11 continued

OR

$$\frac{1}{2}\left(\cot\theta - \frac{1}{\cot\theta}\right) \equiv \frac{\cos\theta}{2\sin\theta} - \frac{\sin\theta}{2\cos\theta}$$ ***A1***

$$\equiv \frac{\cos^2\theta - \sin^2\theta}{2\sin\theta\cos\theta}$$ ***A1***

$$\equiv \frac{\cos 2\theta}{\sin 2\theta}$$ ***A1***

$$\equiv \cot 2\theta$$ ***AG***

[3 marks]

(b) $\frac{1}{2}\left(\cot\theta - \frac{1}{\cot\theta}\right) = \frac{3}{2}\cot\theta\left(\cot^2\theta - 1\right)$

clear denominators (e.g. multiplication by $\cot\theta$) to obtain a quadratic in $\cot^2\theta$ or $\tan^2\theta$: ***(M1)***

$\cot^2\theta - 1 = 3\cot^2\theta\left(\cot^2\theta - 1\right)$ OR $3\cot^4\theta - 4\cot^2\theta + 1 = 0$ OR $3 - 4\tan^2\theta + \tan^4\theta = 0$

attempt to solve a three-term quadratic in $\cot^2\theta$ or $\tan^2\theta$ ***(M1)***

$(3\cot^2\theta - 1)(\cot^2\theta - 1) = 0$, $(3 - \tan^2\theta)(1 - \tan^2\theta) = 0$

$\cot\theta = \pm 1, \pm\frac{1}{\sqrt{3}}$ OR $\tan\theta = \pm 1, \pm\sqrt{3}$ ***(A1)(A1)***

Note: Award ***(A1)(A0)*** for two or three correct values which can be implied by two or three correct values for θ.

$\theta = \frac{\pi}{4}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{3\pi}{4}$ ***A1A1***

Note: Award ***A1A0*** for two or three correct values.
Ignore additional solutions.

Note: Award a maximum of ***(M1)(M0)(A1)(A0)A1A0*** for solutions where $\cot^2\theta - 1$ has been cancelled.

[6 marks]

continued…

Question 11 continued

(c) LHS: $\frac{\mathrm{d}}{\mathrm{d}\theta}\cot(2\theta) = -2\operatorname{cosec}^2 2\theta$ or $-\frac{2}{\sin^2 2\theta}$ ***A1***

EITHER

RHS: attempts to use the chain rule to differentiate $(\cot\theta)^{-1}$ ***(M1)***

$\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\frac{1}{2}\left(\cot\theta - \frac{1}{\cot\theta}\right)\right) = \frac{1}{2}\left(-\operatorname{cosec}^2\theta - \frac{\operatorname{cosec}^2\theta}{\cot^2\theta}\right)$ ***A1A1***

Note: Award ***A1*** for each correct term. Award ***(M0)A1A0*** for only first term correct. These three marks can be awarded independently of first ***A1***.

OR

recognizing that $(\cot\theta)^{-1} = \tan\theta$ ***(M1)***

$\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\frac{1}{2}(\cot\theta - \tan\theta)\right) = \frac{1}{2}\left(-\operatorname{cosec}^2\theta - \sec^2\theta\right)$ or $\frac{1}{2}\left(-\frac{1}{\sin^2\theta} - \frac{1}{\cos^2\theta}\right)$ ***A1A1***

Note: Award ***A1*** for each correct term. Award ***(M0)A1A0*** for only first term correct. These three marks can be awarded independently of first ***A1***.

THEN

$-2\operatorname{cosec}^2 2\theta = \frac{1}{2}\left(-\operatorname{cosec}^2\theta - \sec^2\theta\right)$ or $4\operatorname{cosec}^2 2\theta \equiv \operatorname{cosec}^2\theta + \frac{\operatorname{cosec}^2\theta}{\cot^2\theta}$ or

$\frac{4}{\sin^2 2\theta} \equiv \frac{1}{\sin^2\theta} + \frac{1}{\cos^2\theta}$ ***A1***

$4\operatorname{cosec}^2 2\theta \equiv \operatorname{cosec}^2\theta + \sec^2\theta$ ***AG***

[5 marks]

continued…

Question 11 continued

(d) **METHOD 1**

attempt to use Pythagoras to find PQ ***(M1)***

$\mathrm{PQ}^2=(2\operatorname{cosec}2\theta)^2-(\operatorname{cosec}\theta)^2$

$\mathrm{PQ}=\sec\theta$ or $\mathrm{PQ}^2=\sec^2\theta$ (seen anywhere) ***A1***

$\text{Area}=\frac{1}{2}\operatorname{cosec}\theta\sec\theta=\frac{1}{2\sin\theta\cos\theta}$

attempt to use the double angle formula for $\sin 2\theta$ ***(M1)***

$\text{Area}=\frac{1}{\sin 2\theta}$ or $L=\frac{2}{2\sin\theta\cos\theta}$

$A=\operatorname{cosec}2\theta$ or $A=\frac{1}{\sin 2\theta}$ or $L=\frac{1}{\sin\theta\cos\theta}$ or $L=\sec\theta\operatorname{cosec}\theta$ ***A1***

$L:A=2:1$ ***A1***

[5 marks]

METHOD 2

attempt to find $\mathrm{Q\hat{R}P}$ using right-angled trigonometry ***(M1)***

$\cos\left(\mathrm{Q\hat{R}P}\right)=\frac{\operatorname{cosec}\theta}{2\operatorname{cosec}2\theta}\ \left(=\frac{\sin 2\theta}{2\sin\theta}\right)$

attempt to use the double angle formula for $\sin 2\theta$ ***(M1)***

$\cos\left(\mathrm{Q\hat{R}P}\right)=\frac{2\sin\theta\cos\theta}{2\sin\theta}(=\cos\theta)$

$\mathrm{Q\hat{R}P}=\theta$ ***A1***

$\text{Area}=\frac{1}{2}(2\operatorname{cosec}2\theta)(\operatorname{cosec}\theta)\sin\theta$

$=\operatorname{cosec}2\theta$ ***A1***

$L=2\operatorname{cosec}2\theta$, $A=\operatorname{cosec}2\theta$

$L:A=2:1$ ***A1***

Note: Condone $n=2$ and $\frac{L}{A}=2$ clearly stated

[5 marks]
[Total: 19 marks]

12. (a) $\dfrac{A}{x-1}+\dfrac{B}{3-x}\equiv\dfrac{A(3-x)+B(x-1)}{(3-x)(x-1)}$

$A(3-x)+B(x-1)\equiv 1$ ***(A1)***

EITHER

comparing linear and constant coefficients for numerators ***(M1)***

$A=B,\quad 3A-B=1$

OR

substituting two values of x into $A(3-x)+B(x-1)\equiv 1$ ***(M1)***

$x=3\Rightarrow 2B=1\quad x=1\Rightarrow 2A=1$

THEN

$\left(\dfrac{1}{-3+4x-x^2}\equiv\right)\dfrac{\frac{1}{2}}{x-1}+\dfrac{\frac{1}{2}}{3-x}$ OR $\dfrac{1}{2(x-1)}+\dfrac{1}{2(3-x)}$ ***A1***

Note: Accept $A=\dfrac{1}{2}, B=\dfrac{1}{2}$.

[3 marks]

(b) attempting to separate variables or write as $\dfrac{\mathrm{d}t}{\mathrm{d}N}$ ***(M1)***

$\displaystyle\int\frac{1}{-3+4N-N^2}\mathrm{d}N=\int k\,\mathrm{d}t$, $\dfrac{\mathrm{d}t}{\mathrm{d}N}=\dfrac{1}{k}\left(\dfrac{1}{-3+4N-N^2}\right)$

recognise the need to use partial fractions to integrate ***(M1)***

$\displaystyle\int\frac{1}{2(N-1)}+\frac{1}{2(3-N)}\mathrm{d}N\left(=\int k\,\mathrm{d}t\right)$ or $\displaystyle(t=)\frac{1}{k}\int\frac{1}{2(N-1)}+\frac{1}{2(3-N)}\mathrm{d}N$

$\dfrac{1}{2}\ln(N-1)-\dfrac{1}{2}\ln(3-N)=kt+c$ or equivalent ***A1A1A1***

Note: Award ***A1*** for each term on the LHS and ***A1*** for RHS and constant of integration on either side. Condone presence or absence of modulus signs throughout. Condone missing brackets.

continued…

Question 12 continued

EITHER

$\ln\left(\frac{N-1}{3-N}\right) = 2kt + 2c$ ***A1***

attempt to substitute $t = 0,\ N = 1.5$ ***(M1)***

$\ln\left(\frac{0.5}{1.5}\right) = 2c$ OR $\ln\left(\frac{1}{3}\right) = 2c$

Note: The constant of integration must be present for this ***(M1)*** to be awarded.

OR

attempt to substitute $t = 0,\ N = 1.5$ ***(M1)***

$\frac{1}{2}\ln(0.5) - \frac{1}{2}\ln(1.5) = c$

Note: The constant of integration must be present for this ***(M1)*** to be awarded.

$\frac{1}{2}\ln(N-1) - \frac{1}{2}\ln(3-N) = kt + \frac{1}{2}\ln(0.5) - \frac{1}{2}\ln(1.5)$ or equivalent ***A1***

THEN

$\ln\left(\frac{N-1}{3-N}\right) = 2kt - \ln 3$ ***A1***

$p = 3$

[8 marks]

continued…

Question 12 continued

(c) attempt to exponentiate both sides **(M1)**

$$\frac{N-1}{3-N} = \mathrm{e}^{2kt-\ln 3} \left(\frac{N-1}{3-N} = \mathrm{e}^{2kt-\ln p}\right)$$

$$\frac{N-1}{3-N} = \frac{\mathrm{e}^{2kt}}{3} \left(\frac{N-1}{3-N} = \frac{\mathrm{e}^{2kt}}{p}\right)$$ **(A1)**

attempt to cross multiply and collect terms in N on one side **(M1)**

$$3N + N\mathrm{e}^{2kt} = 3 + 3\mathrm{e}^{2kt} \left(pN + N\mathrm{e}^{2kt} = p + 3\mathrm{e}^{2kt}\right)$$

$$N = \frac{3+3\mathrm{e}^{2kt}}{3+\mathrm{e}^{2kt}} \left(= \frac{p+3\mathrm{e}^{2kt}}{p+\mathrm{e}^{2kt}}\right)$$ **(A1)**

$$N = \frac{3+3\mathrm{e}^{-2kt}}{1+3\mathrm{e}^{-2kt}} \left(= \frac{3+\text{ their } p\mathrm{e}^{-2kt}}{1+\text{ their } p\mathrm{e}^{-2kt}}\right)$$ **A1**

$a = b = c = 3$

> **Note:** For **FT**: $a = 3, b = c =$ their value of p. If their value of p is not an integer or no value of p found award a maximum of ***(M1)(A1)(M1)A1A0.***

[5 marks]

(d) recognition that $\lim_{t\to\infty} \mathrm{e}^{-2kt} = 0$ **(M1)**

$L = 3000$ **A1**

> **Note:** Award ***(M1)A0*** for $L = 3$ or $\left(\lim_{t\to\infty} N =\right)$ (their) a. For ***(M1)A1FT*** their a must be a positive integer and their answer to (c) must be in the required form.

[2 marks]

[Total: 18 marks]